This case reduces immediately to 5., because if $m!$ contains the factors $p_1^{\epsilon_1}, \cdots, p_\nu^{\epsilon_\nu}$, it must also have their product as a factor.

Denoting by max $[\mu(p_1^{\epsilon_1}), \mu(p_2^{\epsilon_\nu}), \cdots, \mu(p_\nu^{\epsilon_\nu})]$ the largest of the numbers $\mu(p_1^{\epsilon_1}), \cdots, \mu(p_\nu^{\epsilon_\nu})$, we have the theorem (which of course includes 3. as a special case):

THEOREM: $\mu(p_1^{\epsilon_1}\cdot p_2^{\epsilon_2}\cdots p_\nu^{\epsilon_\nu}) = \max [\mu(p_1^{\epsilon_1}), \mu(p_2^{\epsilon_2}), \cdots, \mu(p_\nu^{\epsilon_\nu})]$.

Example: $n = 3^2\cdot 5^{29}\cdot 11^{19}\cdot 113$.

$\mu(3^2) = 2\cdot 3 = 6$; $\mu(5^{29}) = 4\cdot 5^2 + 5\cdot 5 = 125$, because $29 = 4(1 + 5) + 5.1$; $\mu(11^{19}) = 1\cdot 11^2 + 7\cdot 11 = 198$, because $19 = 1\cdot(1 + 11) + 7\cdot 1$; $\mu(113) = 113$, because 113 is a prime number; therefore $\mu(3^2\cdot 5^{29}\cdot 11^{19}\cdot 113) = 198$.

Considering $\mu(n)$ as a function of n, one sees that it has, like most number theoretic functions, a very irregular behavior:

$$n = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24,$$

$$\mu(n) = 1, 2, 3, 4, 5, 3, 7, 4, 6, \ \ 5, 11, \ \ 4, 13, \ \ 7, \ \ 5, \ \ 6, 17, \ \ 6, 19, \ \ 5, \ \ 7, 11, 23, \ \ 4.$$

BOOK REVIEWS AND NOTICES.

SEND ALL COMMUNICATIONS ABOUT BOOKS TO W. H. BUSSEY, University of Minnesota.

Descriptive Geometry. By ERVIN KENISON and HARRY CYRUS BRADLEY. The Macmillan Company, New York, 1917. x + 287 pages. \$2.00.

The book under review confines itself to a treatment of that branch of descriptive geometry which is known as the method of double orthographic projection or more simply as the Mongean method.

The very brief introduction which precedes Chapter I is scarcely sufficient to give the reader an adequate notion of the purposes and nature of descriptive geometry. Exception might be taken to the statement: "Its operations are not strictly mathematical." For, aside from the more or less precise mechanical operation involved in the use of the drawing instruments, the methods which descriptive geometry continually applies are, according to Loria, only such as are taught by the old Euclidean geometry and the modern projective geometry, and its processes are so very exact that it is comparable with algebra and analysis.[1]

The first seventeen chapters are concerned principally with the method of representing points, lines, and planes of space, and of solving, by means of plane constructions, the problems of space which involve points, lines, and planes. The great number of problems which have been solved illustrate the more important processes of the Mongean method and give the student a very good notion of the operations of this method. The division of problems into chapters might suggest an attempt at classification. However, no explicit statements have been made which would lead the student to suspect that all the problems

[1] See Loria, *Vorlesungen über Darstellende Geometrie*, preface to Vol. I.

which arise can be divided into a few groups in each of which the solution of a few fundamental ones is sufficient for the solution of all the problems of the group.

In these chapters there are a few inaccuracies to which attention should be called. Thus in Section 34 the statement is made under *case* (*a*) that there is no line in space to correspond to two lines of the picture plane of which one is perpendicular to the ground line. That this statement is not correct can easily be seen if we think of the lines A^h and A^v of Fig. 48 as the traces of planes perpendicular to the horizontal and vertical planes of projection, respectively. These two planes surely intersect in a line, and this is the line represented by the given projections A^h and A^v. Under *case* (*b*) it is stated that there is no line in space corresponding to two lines of the picture plane which are perpendicular to the ground line at different points. Here, again, the lines B^h and B^v of Fig. 49 may be regarded as the traces of planes perpendicular to the horizontal and vertical planes of projection, respectively. These two planes are parallel and hence intersect in a line at infinity.[1] On page 88 the statement is made that a straight line is determined when one of its points and its direction (such as parallel or perpendicular to another line) are known. The words "or perpendicular" should be omitted, for through a point an infinite number of lines can be passed perpendicular to another line.

The last nine chapters deal with curves and surfaces, tangent planes to surfaces, intersections of surfaces by planes, and intersections of surfaces. The surfaces considered are principally cones, cylinders, spheres, and such simple surfaces of revolution as the torus. The problem of developing cylinders and cones is also treated. The lack, or brevity, of definitions in these chapters makes impossible a clear notion of some of the terms used. For instance, no definition is given of a tangent line to a curve, or of a tangent line to a surface. Likewise, the terms surface, curved surface, double curved surface are used without being defined. In Section 163 the statement is made that a double curved surface of revolution is formed by the revolution of any curve about any straight line as an axis, provided the resulting surface is such that no straight line can be drawn upon it. That this definition contains contradictions is evident from the fact that the surface obtained by revolving an hyperbola about its transverse axis is a double curved surface of revolution, and yet it is composed of two one-parameter families of straight lines. To the reviewer it seems that the statement that a general developable surface is composed of the tangents to a space curve is much more helpful to the student than the statement made in Section 179 that every two consecutive elements lie in the same plane. In the chapters on curves and surfaces, as in those on the point, line, and plane, the large number of problems which have been solved give the student a very good notion of the method of procedure.

In addition to the numerous figures giving the Mongean solutions of the various problems considered, the authors have used figures which they call "pictorial representations," such for instance as Figs. 11, 17, 96. They are

[1] For a discussion of these cases, see Loria, *loc. cit.*, Vol. 1, Section 8.

intended to convey to the mind a clearer notion of the space relations than do the Mongean pictures. The authors have not attempted to explain the methods of constructing such figures and there can be no objection to this in a book which concerns itself primarily with the Mongean method. However, such pictures, when used, should be properly drawn. In Fig. 96, for instance, the ellipses representing the circles should not have their principal axes horizontal and vertical, and the lines cc^v and bb^v should be tangent to the upper ellipse and would be if that ellipse were properly drawn. The same remarks apply to the lower ellipse and to certain other figures.

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Plane and Spherical Trigonometry, with Tables. By G. N. BAUER and W. E. BROOKE. Second Revised Edition. D. C. Heath and Co., Boston, 1917. xi + 174 + v + 139 pages.

A writer of textbooks may approach trigonometry from two different standpoints; he may wish to give merely the formulas and processes needed in the solution of triangles, or he may regard trigonometry as a chapter of mathematics dealing with a certain well-defined class of functions whose use in solving triangles is incidental. If in American colleges we could separate our engineering students from the specialists in mathematics, two different types of textbook could be devised to meet these two tendencies; this being as a rule impossible, an attempt must be made to meet both requirements at once. A glance at recent texts shows that we have apparently arrived at a sort of standard in the choice of subject matter; to summarize briefly, an acceptable textbook on trigonometry should contain, it would seem, the following topics:

(*a*) The definitions and elementary relations of the six functions;
(*b*) The addition-formulas and factor-formulas for changing sums of sines or cosines into products;
(*c*) The formulas for $2x$ and $x/2$, or even $3x$, but with no reference to wider formulas of which these might be the simplest cases;
(*d*) The graphs of the functions, either in a separate chapter or throughout;
(*e*) The treatment of trigonometric equations, at least among the exercises, and preferably with ample use of graphic methods;
(*f*) The application of formulas to the solution of plane and spherical triangles.

Besides these absolute essentials, we usually find chapters on

(*g*) The inverse functions;
(*h*) De Moivre's Theorem, and its simpler consequences; such as for instance the formulas for sin (nx), and the definitions of the hyperbolic functions, with their graphs perhaps.

Since trigonometry is usually a 3-hour branch for one semester, these matters can not be handled satisfactorily to any extent; a choice must be made, and here is where the difference in textbooks comes in.